

微分積分「答」

第1章

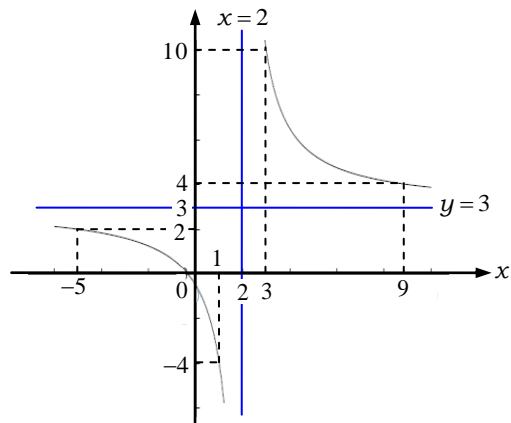
演習 1.1 値域は, (1)  $-7 \leq y \leq -1$ , (2)  $1 \leq y \leq 6$

演習 1.2 (1)  $y = x - 6$  (2)  $y = -2x + 2$

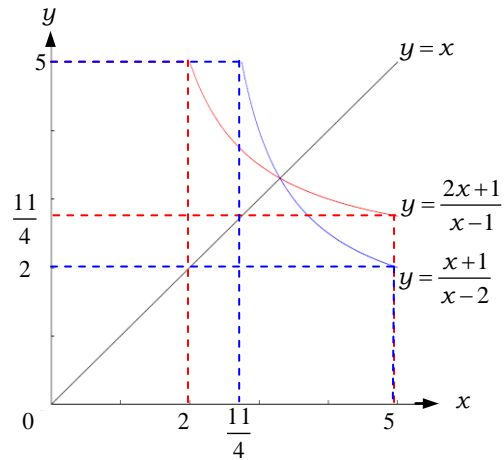
演習 1.3  $x$  軸方向に3,  $y$  軸方向に-5

演習 1.4 (1) 2 (2) 0 (3) 1

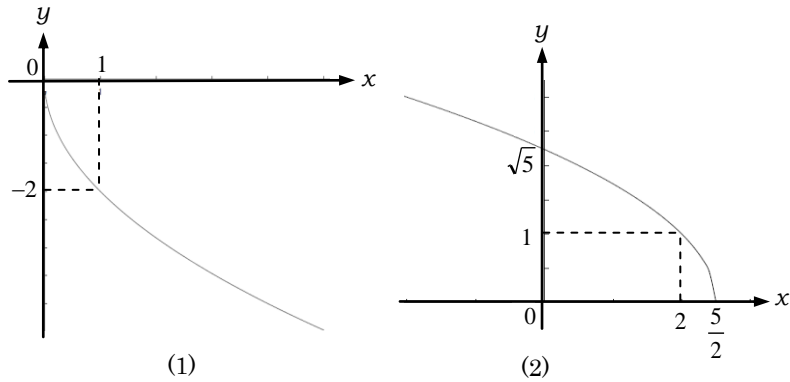
演習 1.5 漸近線:  $x = 2$ ,  $y = 3$



演習 1.6 逆関数:  $y = \frac{x+1}{x-2} \left( \frac{11}{4} \leq x \leq 5 \right)$



- 演習 1.7** (1) 定義域： $x \geq 0$ ，值域： $y \leq 0$   
 (2) 定義域： $x \leq 5/2$ ，值域： $y \geq 0$



- 演習 1.8** (1)  $\frac{\pi}{6}, \frac{5\pi}{6}$  (2)  $\frac{3\pi}{4}, \frac{5\pi}{4}$  (3)  $\frac{2\pi}{3}, \frac{5\pi}{3}$

- 演習 1.9** (1)(a)  $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$  (b)  $0, \frac{\pi}{2}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{3\pi}{2}$

- (2) 最大值  $\sqrt{3}$  ( $x = \frac{\pi}{3}$ ), 最小值  $-\sqrt{3}$  ( $x = \frac{4\pi}{3}$ )

- 演習 1.10** (1)  $-\frac{\pi}{3}$  (2)  $\frac{\pi}{4}$  (3)  $-\frac{\pi}{6}$

演習 1.11 (1)  $-1$  (2)  $x < -1$

演習 1.12 (1)  $4$  (2)  $x > 3$

### 章末問題

1.1  $1 < a < 4$

1.2  $\pi/2$

1.3 最大値:  $\sqrt{2}$  ( $x = y = \pi/4$ )

最小値:  $1$  ( $(x, y) = (0, \pi/2)$  または  $(\pi/2, 0)$ )

1.4 最大値  $81$  ( $x = 2$ )

1.5  $4, 1/8$

### 第2章

演習 2.1 (1)  $\frac{2}{5}$  (2)  $\frac{1}{2}$  (3)  $\frac{1}{4}$  (4)  $3$  (5)  $8$  (6)  $-\infty$

演習 2.2 (1)  $1$  (2)  $2$

演習 2.3 (1)  $1$  (2)  $0$

演習 2.4 (1)  $0$  (2)  $2x$  (3)  $\frac{1}{2\sqrt{x}}$

演習 2.5 (1)  $6x^2 - 6x + 4$  (2)  $8x^3 + 15x^2 + 12x - 1$   
(3)  $\frac{(x-5)(x+1)}{(x-2)^2}$  (4)  $-\frac{4x+9}{(x+1)^6}$

演習 2.6 (1)  $2\cos(2x+1)$  (2)  $\frac{2\sin x}{\cos^3 x}$  (3)  $6\sin^2 2x \cdot \cos 2x$

(4)  $-\frac{2\sin 2x}{(1-\cos 2x)^2}$  (5)  $\frac{2}{(\sin x + \cos x)^2}$

演習 2.7 (1)  $-\frac{1}{\sqrt{1-x^2}}$  (2)  $\frac{1}{1+x^2}$

演習 2.8  $f(x) = \log x$  の  $x=1$  での微分係数を用いる.

演習 2.9 (1)  $\frac{1}{\log 2} \cdot \frac{2x-3}{x^2-3x+1}$  (2)  $\frac{2x-2}{x^2-2x-3}$

$$(3) (3x + \sqrt{2})e^{3x}x^{\sqrt{2}-1} \quad (4) e^{3x}\{3\cos(2x+1) - 2\sin(2x+1)\}$$

$$(5) \frac{1}{\tan x} \quad (6) \frac{1}{\sqrt{x^2+a}}$$

演習 2.10 略

演習 2.11  $\cosh x$

演習 2.12 (1)  $-\frac{b}{a}$  (2)  $\sqrt{3}$  (3)  $-1$

演習 2.13 (1)  $\frac{2(x^3 - 3x^2 - 3x - 3)}{(x^2 + 2x - 1)^3}$  (2) 略 (3)  $-\frac{1}{y^2}$

演習 2.14 (1)  $y_1y = 2p(x + x_1)$  (2)  $y = ex$

### 章末問題

2.1  $-\frac{3}{2}$

2.2 1

2.3 (1)  $-\frac{x+4}{2(x-2)^2\sqrt{x+1}}$  (2)  $(2x^{\pi+1} + \pi x^{\pi-1} + 2x)e^{x^2}$

(3)  $-\frac{1}{\sqrt{x^2-1}}$  (4)  $\frac{2}{\cos^2 x - \sin^2 x}$  (5)  $\frac{3x^2}{\sqrt{1-x^6}}$

(6)  $-\frac{1}{x^2+1}$

2.4 (1)  $\frac{dy}{dx} = -\frac{\sin\theta}{\cos\theta}$ ,  $\frac{d^2y}{dx^2} = \frac{1}{3\cos^4\theta \cdot \sin\theta}$  (2) 1

2.5 接線： $(ax_0 + hy_0)x + (hx_0 + by_0)y = 1$

法線： $(hx_0 + by_0)x - (ax_0 + hy_0)y = h(x_0^2 - y_0^2) - (a-b)x_0y_0$

### 第3章

演習 3.1 (1) 極大值  $f(-1) = 10$ , 極小值  $f(2) = -17$

(2) 極大值  $f\left(\frac{\pi}{4}\right) = \sqrt{2}$

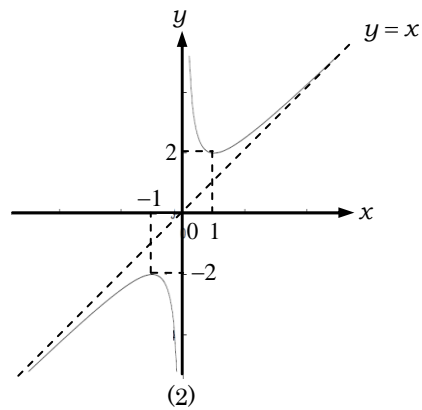
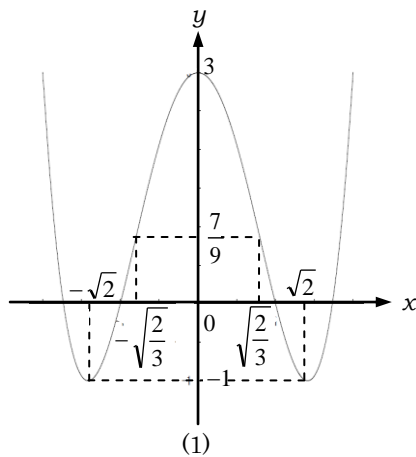
(3) 極小值  $f(0) = 0$ , 極大值  $f(2) = \frac{4}{e^2}$

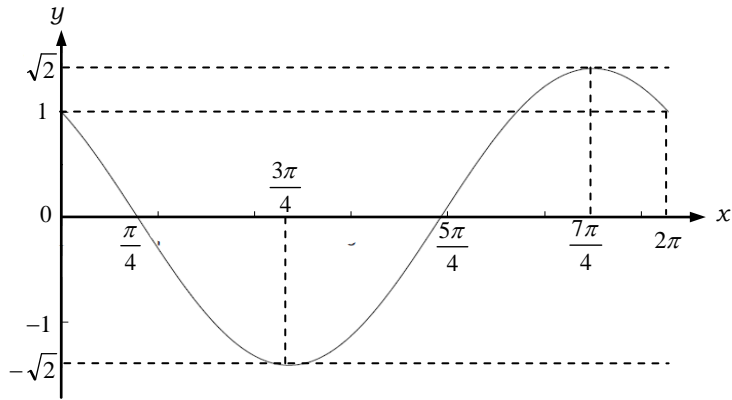
(4) 極小值  $f\left(\frac{\pi}{3\sqrt{3}}\right) = -\frac{\sqrt{3}}{2} e^{-\pi/3\sqrt{3}}$  (5) 極小值  $f\left(\frac{3}{2}\right) = 3\log\frac{3}{2}$

**演習 3.2** (1) 極小值  $f(0) = 1$

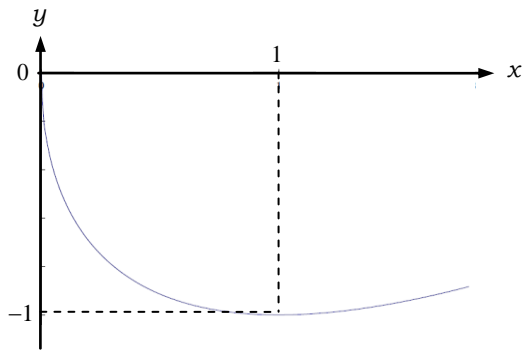
(2) 極大值  $f(0) = 2$ , 極小值  $f\left(-\frac{1}{2}\right) = \frac{7}{4}$ , 極小值  $f\left(\frac{1}{2}\right) = \frac{7}{4}$

**演習 3.3**

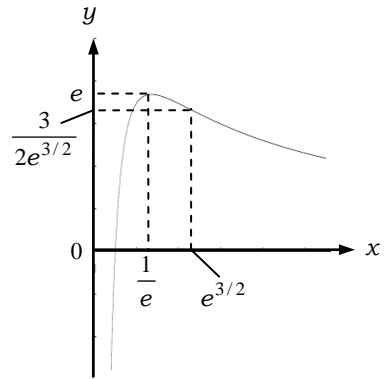




(3)



(4)



(5)

**演習 3.4** (1)  $\ddot{x} = -\omega^2 x$

(2)  $t_1 = \frac{\pi}{2}, \frac{3\pi}{2}, v_m = A, (x_1, y_1) = (0, \pm B),$

$t_2 = 0, \pi, a_m = A, (x_2, y_2) = (\pm A, 0)$

**演習 3.5** 略.

**演習 3.6** (1)  $\frac{1}{6}$  (2) 0

**演習 3.7** (1)  $-2x - 2x^2 + \frac{8}{3}x^3 + \dots$  (2)  $1 - x + \frac{1}{3}x^3 + \dots$

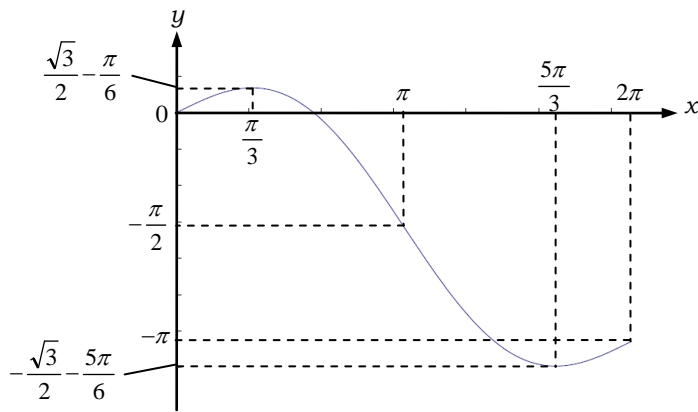
(3)  $x + \frac{1}{6}x^3 + \dots$       (4)  $x - \frac{1}{3}x^3 + \dots$

**演習 3.8** (1)  $-1$  (2)  $-\frac{2}{3}$  (3)  $-\frac{13}{24}$

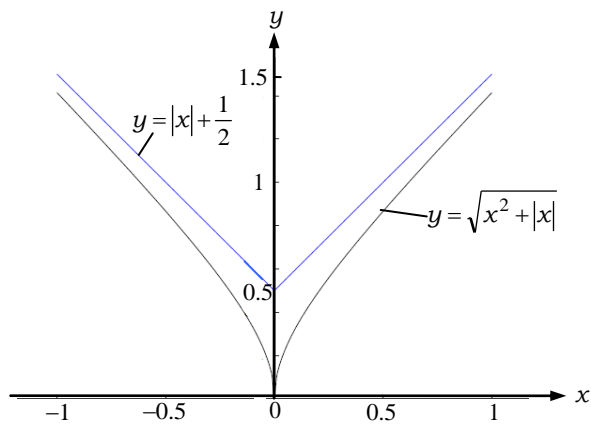
**章末問題**

**3.1** (1) 極大值  $2$ , 極小值  $0$  (2) 極大值  $\frac{3\sqrt{3}}{4}$ , 極小值  $-\frac{3\sqrt{3}}{4}$

**3.2**

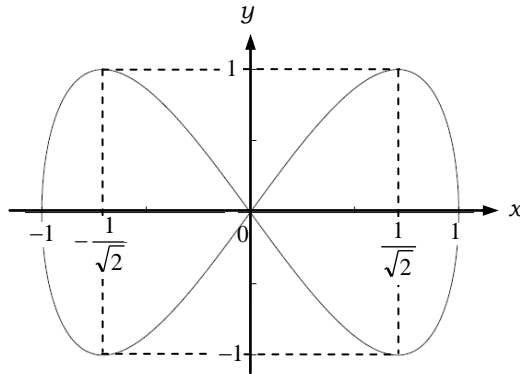


(1)



(2)

3.3 速さ  $v$  の最大値 2, 最小値は  $\frac{\sqrt{31}}{8}$



3.4 (1)  $\frac{H^2 v}{\pi R^2 h^2}$ , (2) 端 A の速さ:  $\omega_0 \sqrt{L^2 - h^2}$ , 端 B の速さ:  $h\omega_0$

3.5 (1)  $\log \frac{3}{2}$  (2) 1

3.6  $\tan x \doteq 1 + 2\varepsilon + 2\varepsilon^2 + \frac{8}{3}\varepsilon^3$ ,  $\tan 46^\circ \doteq 1.0355$

#### 第 4 章

演習 4.1 (1)  $\frac{x^3}{3} + \frac{2}{3}x\sqrt{x} - 2\log|x| + \frac{3}{x} + C$

(2)  $\frac{x^2}{2} + 2x - 3\tan^{-1}x + C$  (3)  $e^{x+2} - e^{x-1} + 2x + C$

(4)  $\tan x - 2x + C$  (5)  $-\cos x + 2\sin^{-1}x + C$

(6)  $x - \tanh x + C$

演習 4.2 (1)  $\frac{1}{3}(2x+3)\sqrt{2x+3}$  (2)  $\frac{2}{3}(x^2-1)\sqrt{x^2-1}$

(3)  $\log|\log x|$  (4)  $\frac{1}{2}\sin^2 x - \frac{2}{3}\sin^3 x$



$$(5) \log|1 + \tan x| \quad (6) \log|\sin x|$$

**演習 4.3** (1)  $-\frac{x}{2} \cos(2x-1) + \frac{1}{4} \sin(2x-1)$

(2)  $\frac{x}{2}(\sin 2x - x \cos 2x) + \frac{1}{4} \cos 2x$

(3)  $\frac{x^2}{2} \left( \log x - \frac{1}{2} \right)$  (4)  $x \sin^{-1} x + \sqrt{1-x^2}$

**演習 4.4** (1)  $\frac{2}{3}(2\sqrt{2}-1) - \frac{1}{2}$  (2)  $\frac{\pi}{4}$  (3)  $-\frac{1}{3}$  (4)  $\frac{1}{\sqrt{3}}$

(5)  $\log \frac{e+e^{-1}}{2}$  (6)  $\log \frac{4\sqrt{3}}{3}$

**演習 4.5** (1)  $-\frac{2\sqrt{2}-1}{3}$  (2)  $\frac{\pi}{4}$  (3) 2

**演習 4.6** (1)  $-2\pi$  (2)  $\frac{1}{2}$  (3)  $\frac{\pi}{4} - \frac{1}{2}$  (4)  $\frac{1}{27}(5e^3 - 2)$

(5)  $\frac{2}{5}(1 + e^{-\pi/2})$

**演習 4.7** (1)  $\log \left| \frac{x-1}{x} \right|$  (2)  $\frac{3}{2} + \frac{1}{e+1} + 2 \log \frac{2}{e+1}$

**演習 4.8** (1)  $\frac{1}{2} \log \frac{1+\sin x}{1-\sin x}$  (2)  $\frac{8}{15}$

(3)  $n$  が偶数のとき,  $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$ ,

$n$  が奇数のとき,  $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{2}{3} \cdot 1$

**演習 4.9** (1)  $\log \left| \tan \frac{x}{2} \right|$  (2)  $\log \left| \frac{1 + \tan \frac{x}{2}}{1 - \tan \frac{x}{2}} \right|$

$$(3) \frac{2}{1 - \tan \frac{x}{2}} \quad (4) \sqrt{2} \log(\sqrt{2} + 1)$$

**演習 4.10** 略.

**演習 4.11** (1)  $2xe^{x^2} \cdot \cos 2x^2$  (2)  $1 - \cos x$

(3)  $x \log|x| - \log|2x-1| + x - 1$

**演習 4.12** (1)  $1/4$  (2)  $\pi/4$  (3)  $4/\pi^2$

### 章末問題

**4.1** (1)  $2\sqrt{x}(\log x - 2)$  (2)  $\frac{1}{\sqrt{5}} \log \left| \frac{\sqrt{5} \tan \frac{t}{2} - 1}{\sqrt{5} \tan \frac{x}{2} + 1} \right|$

**4.2** 略

**4.3** (1)  $8$  (2)  $\frac{\pi^2}{4}$

**4.4** (1)  $1$  (2)  $1$

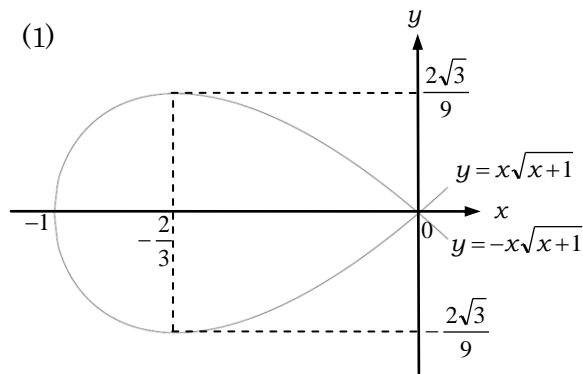
**4.5** 略

### 第5章

**演習 5.1** (1)  $\frac{1}{2}(e^2 - 1)$  (2)  $\frac{3}{2} \log 2$  (3)  $\frac{2}{e}$

**演習 5.2** (1)  $\frac{1}{6}$  (2)  $e^2 - 5$  (3)  $\frac{9\sqrt{3}}{4} - 1$

**演習 5.3**



(2)  $\frac{8}{15}$

演習 5.4  $\frac{3\pi}{8}$

演習 5.5 (1)  $\frac{\pi^2}{2}$  (2)  $\pi(e-2)$  (3)  $\frac{32}{105}\pi$

演習 5.6  $\frac{\sqrt{3}}{16}(4-\pi)$

演習 5.7 (1)  $\frac{2}{3}(2\sqrt{2}-1)$  (2)  $3+\frac{1}{8}\log 2$

演習 5.8 6

演習 5.9  $6\sqrt{3}$

演習 5.10 (1)  $\frac{\pi}{4}(e^2 - e^{-2} + 4)$  (2)  $\frac{6\pi}{5}$

演習 5.11 (1) -1 (2)  $\pi$  (3) 4

演習 5.12 (1)  $\frac{\pi}{8}$  (2)  $\infty$  (3)  $\frac{1}{2}$

章末問題

5.1 (1)  $\frac{9}{2}$  (2)  $3e^{2/3}-5$  (3)  $\frac{1}{4}$  (4)  $\frac{8}{3}\sqrt{2}$

5.2  $\frac{\pi^2}{12}(2\pi^2-3)$

$$5.3 \quad 1 + \frac{1}{\sqrt{2}} \log(1 + \sqrt{2})$$

$$5.4 \quad 2\pi ab \left( \sqrt{1 - e^2} + \frac{1}{e} \sin^{-1} e \right)$$

$$5.5 \quad \frac{\pi}{4\sqrt{2}} \log 2$$

### 第6章

**演習 6.1**  $f(1, 1) = 3, \quad f(1, 3) = 17, \quad f(-1, 2) = -3$

**演習 6.2** (1) 不連続 (2) 連続

**演習 6.3** (1)  $f_x = \frac{2y}{(x+y)^2}, \quad f_y = \frac{-2x}{(x+y)^2}$

(2)  $f_x = e^{-(x+y)} \{ \cos(x-y) - \sin(x-y) \}$

$$f_y = -e^{-(x+y)} \{ \cos(x-y) + \sin(x-y) \}$$

(3)  $f_x = y \cdot x^{y-1}, \quad f_y = x^y \cdot \log x$

(4)  $f_x = 2x \tan^{-1}(x-y) + \frac{x^2 + y^2}{1 + (x-y)^2},$

$$f_y = 2y \tan^{-1}(x-y) - \frac{x^2 + y^2}{1 + (x-y)^2}$$

(5)  $f_x = -\frac{\log y}{x(\log x)^2}, \quad f_y = \frac{1}{y \log x}$

**演習 6.4**  $z_x = -\frac{x}{r^3}, \quad z_y = -\frac{y}{r^3}$

**演習 6.5** (1)  $f_x(0, 0) = -3, \quad f_y(0, 0) = 0$

(2)  $f_x(0, 0)$  は存在しない,  $f_y(0, 0) = 0$

**演習 6.6** (1)  $f_{xx} = -\frac{2y^2}{(x+y)^3}$ ,  $f_{xy} = f_{yx} = \frac{2xy}{(x+y)^3}$ ,  $f_{yy} = -\frac{2x^2}{(x+y)^3}$

(2)  $f_{xx} = -\frac{2(x-y)}{\{1+(x-y)^2\}^2}$ ,  $f_{xy} = f_{yx} = \frac{2(x-y)}{\{1+(x-y)^2\}^2}$   
 $f_{yy} = \frac{2(x-y)}{\{1+(x-y)^2\}^2}$

**演習 6.7** (1)  $(2xy + y^3)dx + (x^2 + 3xy^2)dy$

(2)  $\left(\cos x + \frac{1}{\sqrt{y^2 - x^2}}\right)dx - \left(x \sin y + \frac{x}{y\sqrt{y^2 - x^2}}\right)dy$

**演習 6.8** (1)  $-\frac{1}{\sin^2 t}$  (2)  $-\frac{1}{t}$

**演習 6.9** 略.

**演習 6.10** (1)  $f_u = \frac{2(uv^2 + u + v)}{u^2v^2 + (u+v)^2}$ ,  $f_v = \frac{2(u^2v + u + v)}{u^2v^2 + (u+v)^2}$

(2)  $f_r^2 + \frac{1}{r^2}f_\theta^2 + \frac{1}{r^2 \sin^2 \theta}f_\phi^2$

**演習 6.11** (1)  $y' = \frac{x+1}{2y+1}$ ,  $y'' = \frac{1-2y'^2}{2y+1}$

(2)  $y' = \frac{z-x}{y-z}$ ,  $z' = \frac{y-x}{z-y}$

**演習 6.12**  $-\frac{\pi}{2}(x-1) - \left(y - \frac{\pi}{2}\right) - (x-1)\left(y - \frac{\pi}{2}\right)$

**演習 6.13** (1) 極小値  $f(1, 0) = -1$  (2)  $f(x, y)$  に極値は存在しない

(3) 極大値  $f(\pi/2, \pi/2) = 3$

### 章末問題

**6.1** (1) 極限值は存在しない. (2) 不連続

**6.2** (1)  $f_x = x^y y^x \cdot \left(\frac{y}{x} + \log y\right)$ ,  $f_y = x^y y^x \cdot \left(\frac{x}{y} + \log x\right)$

(2)  $f_x = \frac{x}{x^2 + y^2}$ ,  $f_y = \frac{y}{x^2 + y^2}$

**6.3** (1)  $\frac{1}{r}$  (2)(i) 略 (ii)  $-f(r)$

**6.4** 略.

**6.5** (1) 極大値  $f(0, 0) = 1$

(2)  $(x, y) = \left(\frac{1}{2}, \frac{1}{2}\right)$  のとき, 極大値  $z = \frac{1 + \sqrt{3}}{2}$ , 極小値  $z = \frac{1 - \sqrt{3}}{2}$

## 第7章

**演習 7.1** (1)  $\frac{2}{3}$  (2)  $(\log 2)^2$  (3) 1 (4)  $\pi - 2$

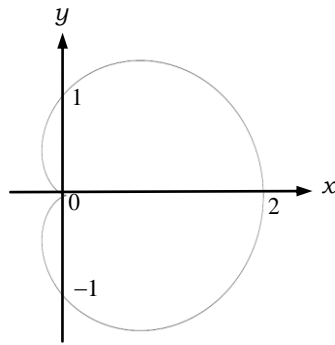
**演習 7.2** (1)  $\frac{4}{27}$  (2)  $\frac{1}{2}e(e^2 - 3) + 1$

**演習 7.3** (1)  $\frac{4}{9}$  (2)  $\frac{e-2}{2}$

**演習 7.4** (1)  $\left(-\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right)$  (2)  $\left(\frac{4\sqrt{3}}{3}, \frac{5}{6}\pi\right)$

**演習 7.5** (1)  $x + \sqrt{3}y = 2$ , 2点  $(2, 0)$ ,  $\left(0, \frac{2\sqrt{3}}{3}\right)$  を通る直線

(2)  $r = 1 + \cos \theta$



$$(3) \quad r = \frac{el}{1 + e \cos \phi}$$

$0 < e < 1$  のとき,  $(1 - e^2)x^2 + 2e^2lx + y^2 = e^2l^2$  : 楕円,

$e = 1$  のとき,  $y^2 = l^2 - 2lx$  : 放物線

**演習 7.6** (1)  $\frac{14}{3}\pi$  (2)  $\frac{2}{15}$

**演習 7.7** (1) 4 (2)  $\frac{\sqrt{\pi}}{4}$

**演習 7.8** (1)  $\frac{255}{32}\pi$  (2)  $\frac{7}{9}\pi$

**演習 7.9** (1)  $\frac{2}{105}$  (2)  $2\pi\left(1 - \frac{3}{4}\log 3\right)$

**演習 7.10** (1)  $\frac{1}{3}$  (2)  $\frac{3}{4}a$

**演習 7.11** (1)  $(X, Y) = \left(\frac{1}{5}, \frac{1}{5}\right)$  (2)  $\frac{5}{6}$

**章末問題**

**7.1** (1)  $\frac{4}{3}$  (2)  $2\log 2 - \frac{3}{2}$  (3)  $\frac{27}{10}$

**7.2** (1)  $\frac{1}{4}$  (2)  $\frac{1}{24}$

**7.3** (1)  $\frac{5}{4}\pi$  (2)  $\frac{35}{16}\pi$  (3)  $\frac{8}{5}\pi$

**7.4** (1)  $\frac{2}{15}$  (2)  $4\pi$  (3)  $\frac{16}{3}\left(\pi - \frac{4}{3}\right)$

**7.5** (1)  $\left(\frac{\pi}{2} - 1, \frac{\pi}{8}\right)$  (2)  $\frac{3 \sin \beta}{4 \beta} \alpha$  (3)  $\left(\frac{3}{8}, \frac{9}{64}\pi, \frac{9}{64}\pi\right)$